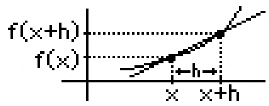
2a (11-97)

Calcu-List



$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Vertical Motion in Feet and Seconds:

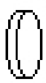
$a(t) = v'(t) = h''(t)$

$h(t) = -16t^2 + V_0 t + H_0$

$v(t) = -32t + V_0$

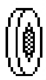
$a(t) = -32$

Volumes of Rotation



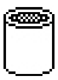
Discs

$\pi \int r^2 \, dx$



Washers

$\pi \int R^2 - r^2 \, dx$



Shells

$2\pi \int r\,h \, dx$

The Fundamental Theorem of Calculus!

If $f'(x)$ is continuous from a to b then:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

If $f(x)$ is continuous from a to b then:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} \quad \text{or} \quad \frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$$

Graphing Tips

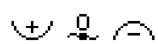
$\lim_{x \to \pm\infty} f(x) = c$ → Horizontal Asymptote at $y = c$

$\lim_{x \to \pm\infty} f(x) = cx$ → Slant Asymptote with slope c

f(undefined value) $= \frac{c}{0}$ → Vertical Asymptote

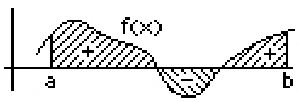
f(undefined value) $= \frac{0}{0}$ → Hole in the graph

y' = slope → 

y'' = concavity → 

y' = 0 or Ø → Indicates possible Max or Min

y'' = 0 or Ø → Indicates possible Inflection Point



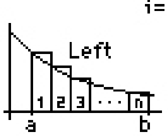
$$\text{Net Area} = \int_a^b f(x)\,dx$$

Trapezoidal Rule (n is the number of trapezoids)

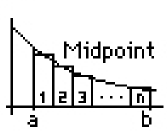
$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + f(x_n)\right)$$

Approximate Area Using Rectangles of Equal Width

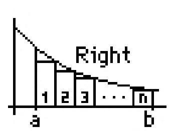
$$\text{Area} = \sum_{i=1}^{n} f(c_i)\cdot\Delta x \qquad \Delta x = \frac{b-a}{n}$$



$c_i = a + (i-1)\cdot\Delta x$



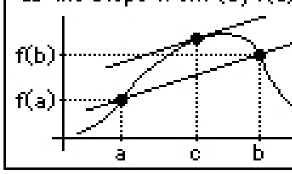
$c_i = a + (i-\frac{1}{2})\cdot\Delta x$



$c_i = a + i\cdot\Delta x$

$\frac{d}{dx}(ax^n) = nax^{n-1}$	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}(uv) = u'v + uv'$	$\int e^x \, dx = e^x + c$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$	$\int \frac{1}{x} \, dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}(e^x) = e^x$	$\int \sin x \, dx = -\cos x + c$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \cos x \, dx = \sin x + c$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert + c$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec x \, dx = -\ln\lvert\sec x - \tan x\rvert + c$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc x \, dx = \ln\lvert\csc x - \cot x\rvert + c$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec^2 x \, dx = \tan x + c$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc^2 x \, dx = -\cot x + c$
$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$	$\int \sec x \tan x \, dx = \sec x + c$
$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$	$\int \csc x \cot x \, dx = -\csc x + c$
$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$	$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + c$
$\frac{d}{dx}(\text{arccot } x) = \frac{-1}{1+x^2}$	$\int \frac{1}{1+x^2} \, dx = \arctan x + c$
$\frac{d}{dx}(\text{arcsec } x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \text{arcsec}\lvert x\rvert + c$
$\frac{d}{dx}(\text{arccsc } x) = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	

If $f(x)$ is continuous and differentiable from a to b, then there is a x-value c such that the slope at c is the same as the slope from $(a, f(a))$ to $(b, f(b))$.



The Mean Value Theorem

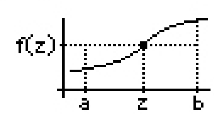
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

From a to b on a continuous $f(x)$ there is a z such that:

- At z, $f(x)$ takes on the average value.
- $f(z)$ is the average value.

Average Value

$$f(z) = \frac{\int_a^b f(x)\,dx}{b-a}$$



Separable Differential Equations -- Exponential Growth

When y is directly proportional to the rate at which y changes: $\Rightarrow \frac{dy}{dt} = ry$

$$\Rightarrow \frac{1}{y}\,dy = r\,dt \Rightarrow \int \frac{1}{y}\,dy = \int r\,dt \Rightarrow \ln y = rt + c$$

$$\Rightarrow e^{\ln y} = e^{rt+c} \Rightarrow y = e^{rt}\cdot e^{c} \Rightarrow \boxed{y = p\,e^{rt}}$$